1. Let S = {a, b, c, d, e, f, g} be a collection of items with weight-benefit values as follows: a(3, 12),

b(6, 12), c(6, 9), d(1, 5), e(2, 5), f(10, 10), g(3, 9). For example, item a weighs 3 lbs and is worth a total

of $12. Assume you have a knapsack that can hold a total of 11 lbs.

(a) (10 points) What is an optimal solution to the fractional knapsack problem for S? Show your

work.

With the values above: a(3, 12), b(6, 12), c(6, 9), d(1, 5), e(2, 5), f(10, 10), g(3, 9)

the items benefit per weight values are: a(4), b(2), c(1.5), d(5), e(2.5), f(1), g(3)

and a total knapsack weight of 11lbs:

first take all of item d which is 1 lb, knapsack now has{1 lb of d} total weight is 1 lb total value is $5

then take all of a which is 3 lbs, knapsack now has{1 lb of d, 3 lbs of a} total weight is 4 lbs total value is $17

next take all of g which is 3 lbs, knapsack now has{1 lb of d, 3 lbs of a, 3 lbs of g} total weight is 7 lbs total value is $26

next take all of e which is 2 lbs, knapsack now has{1 lb of d, 3 lbs of a, 3 lbs of g, 2 lbs of e} total weight is 9 lbs total value is $31

finally take 2 lbs of b, knapsack now has{1 lb of d, 3 lbs of a, 3 lbs of g, 2 lbs of e, 2 lbs of b} total weight is 11 lbs total value is $35

(b) (10 points) What is an optimal solution to the 0-1 knapsack problem for S? Show your work.

With the values above: a(3, 12), b(6, 12), c(6, 9), d(1, 5), e(2, 5), f(10, 10), g(3, 9)

the items benefit per weight values are: a(4), b(2), c(1.5), d(5), e(2.5), f(1), g(3)

and a total knapsack weight of 11lbs:

first take all of item d which is 1 lb, knapsack now has{1 lb of d} total weight is 1 lb total value is $5

then take all of a which is 3 lbs, knapsack now has{1 lb of d, 3 lbs of a} total weight is 4 lbs total value is $17

next take all of g which is 3 lbs, knapsack now has{1 lb of d, 3 lbs of a, 3 lbs of g} total weight is 7 lbs total value is $26

finally take all of e which is 2 lbs, knapsack now has{1 lb of d, 3 lbs of a, 3 lbs of g, 2 lbs of e} total weight is 9 lbs total value is $31

This is the most we can fit in our knapsack given that we take all or none of a given item.

2. (5 points) Suppose we are given a set of tasks specified by pairs of start times and finish times

as T = {(5, 6),(9, 11)(3, 7),(1, 2),(10, 12),(6, 8),(1, 3),(7, 9),(1, 4),(11, 14),(2, 5),(4, 9),(7, 10)}. Solve

the task scheduling problem for these tasks.

Beginning with no tasks currently scheduled:

first we’ll order our list by start times:

T={(1, 2), (1, 3), (1, 4), (2, 5), (3, 7), (4, 9), (5, 6), (6, 8), (7, 9), (7, 10), (9, 11), (10, 12), (11, 14)}

we”ll assign our first task (1,2)

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1:00 | 2:00 | 3:00 | 4:00 | 5:00 | 6:00 | 7:00 | 8:00 | 9:00 | 10:00 | 11:00 | 12:00 |
| Machine 7 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 6 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 5 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 3 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 2 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 1 | (1,2) |  |  |  |  |  |  |  |  |  |  |  |

next we’ll assign the next task in T (1,3)

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1:00 | 2:00 | 3:00 | 4:00 | 5:00 | 6:00 | 7:00 | 8:00 | 9:00 | 10:00 | 11:00 | 12:00 |
| Machine 7 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 6 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 5 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 3 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 2 | (1,3) | (1,3) |  |  |  |  |  |  |  |  |  |  |
| Machine 1 | (1,2) |  |  |  |  |  |  |  |  |  |  |  |

next we’ll assign the next task in T (1,4)

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1:00 | 2:00 | 3:00 | 4:00 | 5:00 | 6:00 | 7:00 | 8:00 | 9:00 | 10:00 | 11:00 | 12:00 |
| Machine 7 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 6 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 5 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 3 | (1,4) | (1,4) | (1,4) |  |  |  |  |  |  |  |  |  |
| Machine 2 | (1,3) | (1,3) |  |  |  |  |  |  |  |  |  |  |
| Machine 1 | (1,2) |  |  |  |  |  |  |  |  |  |  |  |

next we’ll assign the next task in T (2, 5)

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1:00 | 2:00 | 3:00 | 4:00 | 5:00 | 6:00 | 7:00 | 8:00 | 9:00 | 10:00 | 11:00 | 12:00 |
| Machine 7 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 6 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 5 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 3 | (1,4) | (1,4) | (1,4) |  |  |  |  |  |  |  |  |  |
| Machine 2 | (1,3) | (1,3) |  |  |  |  |  |  |  |  |  |  |
| Machine 1 | (1,2) | (2, 5) | (2, 5) | (2, 5) |  |  |  |  |  |  |  |  |

next we’ll assign the next task in T (3, 7)

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1:00 | 2:00 | 3:00 | 4:00 | 5:00 | 6:00 | 7:00 | 8:00 | 9:00 | 10:00 | 11:00 | 12:00 |
| Machine 7 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 6 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 5 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 3 | (1,4) | (1,4) | (1,4) |  |  |  |  |  |  |  |  |  |
| Machine 2 | (1,3) | (1,3) | (3, 7) | (3, 7) | (3, 7) | (3, 7) |  |  |  |  |  |  |
| Machine 1 | (1,2) | (2, 5) | (2, 5) | (2, 5) |  |  |  |  |  |  |  |  |

next we’ll assign the next task in T (4, 9)

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1:00 | 2:00 | 3:00 | 4:00 | 5:00 | 6:00 | 7:00 | 8:00 | 9:00 | 10:00 | 11:00 | 12:00 |
| Machine 7 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 6 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 5 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 3 | (1,4) | (1,4) | (1,4) | (4, 9) | (4, 9) | (4, 9) | (4, 9) | (4, 9) |  |  |  |  |
| Machine 2 | (1,3) | (1,3) | (3, 7) | (3, 7) | (3, 7) | (3, 7) |  |  |  |  |  |  |
| Machine 1 | (1,2) | (2, 5) | (2, 5) | (2, 5) |  |  |  |  |  |  |  |  |

next we’ll assign the next task in T (5, 6)

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1:00 | 2:00 | 3:00 | 4:00 | 5:00 | 6:00 | 7:00 | 8:00 | 9:00 | 10:00 | 11:00 | 12:00 |
| Machine 7 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 6 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 5 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 3 | (1,4) | (1,4) | (1,4) | (4, 9) | (4, 9) | (4, 9) | (4, 9) | (4, 9) |  |  |  |  |
| Machine 2 | (1,3) | (1,3) | (3, 7) | (3, 7) | (3, 7) | (3, 7) |  |  |  |  |  |  |
| Machine 1 | (1,2) | (2, 5) | (2, 5) | (2, 5) | (5, 6) |  |  |  |  |  |  |  |

next we’ll assign the next task in T (6, 8)

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1:00 | 2:00 | 3:00 | 4:00 | 5:00 | 6:00 | 7:00 | 8:00 | 9:00 | 10:00 | 11:00 | 12:00 |
| Machine 7 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 6 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 5 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 3 | (1,4) | (1,4) | (1,4) | (4, 9) | (4, 9) | (4, 9) | (4, 9) | (4, 9) |  |  |  |  |
| Machine 2 | (1,3) | (1,3) | (3, 7) | (3, 7) | (3, 7) | (3, 7) |  |  |  |  |  |  |
| Machine 1 | (1,2) | (2, 5) | (2, 5) | (2, 5) | (5, 6) | (6, 8) | (6, 8) |  |  |  |  |  |

next we’ll assign the next task in T (7, 9)

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1:00 | 2:00 | 3:00 | 4:00 | 5:00 | 6:00 | 7:00 | 8:00 | 9:00 | 10:00 | 11:00 | 12:00 |
| Machine 7 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 6 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 5 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 3 | (1,4) | (1,4) | (1,4) | (4, 9) | (4, 9) | (4, 9) | (4, 9) | (4, 9) |  |  |  |  |
| Machine 2 | (1,3) | (1,3) | (3, 7) | (3, 7) | (3, 7) | (3, 7) | (7, 9) | (7, 9) |  |  |  |  |
| Machine 1 | (1,2) | (2, 5) | (2, 5) | (2, 5) | (5, 6) | (6, 8) | (6, 8) |  |  |  |  |  |

next we’ll assign the next task in T (7, 10)

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1:00 | 2:00 | 3:00 | 4:00 | 5:00 | 6:00 | 7:00 | 8:00 | 9:00 | 10:00 | 11:00 | 12:00 |
| Machine 7 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 6 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 5 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  | (7, 10) | (7, 10) | (7, 10) |  |  |  |
| Machine 3 | (1,4) | (1,4) | (1,4) | (4, 9) | (4, 9) | (4, 9) | (4, 9) | (4, 9) |  |  |  |  |
| Machine 2 | (1,3) | (1,3) | (3, 7) | (3, 7) | (3, 7) | (3, 7) | (7, 9) | (7, 9) |  |  |  |  |
| Machine 1 | (1,2) | (2, 5) | (2, 5) | (2, 5) | (5, 6) | (6, 8) | (6, 8) |  |  |  |  |  |

next we’ll assign the next task in T (9, 11)

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1:00 | 2:00 | 3:00 | 4:00 | 5:00 | 6:00 | 7:00 | 8:00 | 9:00 | 10:00 | 11:00 | 12:00 |
| Machine 7 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 6 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 5 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  | (7, 10) | (7, 10) | (7, 10) |  |  |  |
| Machine 3 | (1,4) | (1,4) | (1,4) | (4, 9) | (4, 9) | (4, 9) | (4, 9) | (4, 9) |  |  |  |  |
| Machine 2 | (1,3) | (1,3) | (3, 7) | (3, 7) | (3, 7) | (3, 7) | (7, 9) | (7, 9) |  |  |  |  |
| Machine 1 | (1,2) | (2, 5) | (2, 5) | (2, 5) | (5, 6) | (6, 8) | (6, 8) |  | (9, 11) | (9, 11) |  |  |

next we’ll assign the next task in T (10, 12)

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1:00 | 2:00 | 3:00 | 4:00 | 5:00 | 6:00 | 7:00 | 8:00 | 9:00 | 10:00 | 11:00 | 12:00 |
| Machine 7 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 6 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 5 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  | (7, 10) | (7, 10) | (7, 10) |  |  |  |
| Machine 3 | (1,4) | (1,4) | (1,4) | (4, 9) | (4, 9) | (4, 9) | (4, 9) | (4, 9) |  |  |  |  |
| Machine 2 | (1,3) | (1,3) | (3, 7) | (3, 7) | (3, 7) | (3, 7) | (7, 9) | (7, 9) |  | (10, 12) | (10, 12) |  |
| Machine 1 | (1,2) | (2, 5) | (2, 5) | (2, 5) | (5, 6) | (6, 8) | (6, 8) |  | (9, 11) | (9, 11) |  |  |

next we’ll assign the next task in T (11, 14)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1:00 | 2:00 | 3:00 | 4:00 | 5:00 | 6:00 | 7:00 | 8:00 | 9:00 | 10:00 | 11:00 | 12:00 | 13:00 |
| Machine 7 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 6 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 5 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Machine 4 |  |  |  |  |  |  | (7, 10) | (7, 10) | (7, 10) |  |  |  |  |
| Machine 3 | (1,4) | (1,4) | (1,4) | (4, 9) | (4, 9) | (4, 9) | (4, 9) | (4, 9) |  |  | (11, 14) | (11, 14) | (11, 14) |
| Machine 2 | (1,3) | (1,3) | (3, 7) | (3, 7) | (3, 7) | (3, 7) | (7, 9) | (7, 9) |  | (10, 12) | (10, 12) |  |  |
| Machine 1 | (1,2) | (2, 5) | (2, 5) | (2, 5) | (5, 6) | (6, 8) | (6, 8) |  | (9, 11) | (9, 11) |  |  |  |

3. Consider the single machine scheduling problem where we are given a set T of tasks specified by their

start times and finish times, as in the task scheduling problem, except now we have only one machine

and we wish to maximize the number of tasks that this single machine performs.

(a) (7 points) Design a greedy algorithm for this single machine scheduling problem. What is the

running time of your algorithm?

Algorithm schedule(set of tasks T with n tasks):

input: a set of tasks T with n tasks

output: an array with our scheduled tasks

sort(T based on finish times in ascending order)

int j=0

for(int i=0; i<n-1;)

if(i==0 && j==0)

X[i]=T[j]

++j

++i

else if(T[j]->beginTime() >= X[i-1]->endTime())

X[i]=T[j]

++j

++i

else

++j

return X

O(n log(n)) do to using sort

(b) (3 points) Prove the correctness of your algorithm.

Given T = {(5, 6),(9, 11)(3, 7),(1, 2),(10, 12),(6, 8),(1, 3),(7, 9),(1, 4),(11, 14),(2, 5),(4, 9),(7, 10)}

Algorithm schedule(set of tasks T with n tasks):

input: a set of tasks T with n tasks

output: an array with our scheduled tasks

sort(T based on finish times in ascending order)

T={(1, 2), (1, 3), (1, 4), (2, 5), (5, 6), (3, 7), (6, 8), (4, 9),( 7, 9), (7, 10), (9, 11), (10, 12), (11, 14)}

int j=0

for(int i=0; i<n-1;)

if(i==0 && j==0)

X[i]=T[j]

++j

++i

else if(T[j]->beginTime() >= X[i-1]->endTime())

X[i]=T[j]

++j

++i

else

++j

return X

Output: X={(1,2), (2, 5), (5, 6), (6, 8), (9, 11), (11, 14)}

4. (10 points) For each of the following recurrence equations which describe the running time T(n) of

a recursive algorithm, use the master method to express the asymptotic complexity (assuming that

T(n) = c for n < d, for constants c > 0 and d ≥ 1).

(a) T(n) = 2T(n/2) + log n

logba=1

f(n)=log(n)

so, case 1 says T(n) is big theta of n

(b) T(n) = 8T(n/2) + n2

logba=3

f(n)=n2

so, case 1 says T(n) is big theta of n3

(c) T(n) = 7T(n/3) + n

logba= 1.7712437492

f(n)=n

so, case 1 says T(n) is big theta of n1.7712437492

(d) T(n) = 4T(n/2) + n2

logba= 2

f(n)=n2

so, case 2 says T(n) is big theta of n2 log(n) n2

(e) T(n) = 3T(n/2) + n2

logba= 1.5849625007

f(n)= n2

so, case 3 says T(n) is big theta of n2

5. (10 points) What is the best way to multiply a chain of matrices with dimensions that are 10 × 5,

5 × 2, 2 × 20, 20 × 12, 12 × 4, and 4 × 60? Show your work (including two tables - one indicating the

index k which gives the final multiply index for each subproblem, and the second table which indicates

the optimal number of multiplications for each subproblem).

A0 A1 A2 A3 A4 A5

Using the set A={(10 × 5), (5 × 2), (2 × 20), (20 × 12), (12 × 4), (4 × 60)}

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| N | 0 | 1 | 2 | 3 | 4 | 5 |
| 0 | 0 | 100 | 500 | 820 | 756 | 2356 |
| 1 |  | 0 | 200 | 600 | 616 | 1656 |
| 2 |  |  | 0 | 480 | 576 | 1056 |
| 3 |  |  |  | 0 | 960 | 5760 |
| 4 |  |  |  |  | 0 | 2880 |
| 5 |  |  |  |  |  | 0 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| k | 0 | 1 | 2 | 3 | 4 | 5 |
| 0 |  | 0 | 1 | 1 | 1 | 1 |
| 1 |  |  | 1 | 1 | 1 | 1 |
| 2 |  |  |  | 2 | 3 | 4 |
| 3 |  |  |  |  | 3 | 4 |
| 4 |  |  |  |  |  | 4 |
| 5 |  |  |  |  |  |  |

A0\*A1 N[0][1]=0+0+10\*5\*2=100 A1\*A2 N[1][2]=0+0+5\*2\*20=200

A2\*A3 N[2][3]=0+0+2\*20\*12=480 A3\*A4 N[3][4]=0+0+20\*12\*4=960

A4\*A5 N[4][5]=0+0+12\*4\*60=2880

(A0)(A1\*A2) = 0+200+10\*5\*20=1200 (A0\*A1)(A2)=100+0+10\*2\*20=500

(A1)(A2\*A3)=0+480+5\*2\*12=600 (A1\*A2)(A3)=200+0+5\*20\*12=1400

(A2)(A3\*A4)=0+960+2\*20\*4=1120 (A2\*A3)(A4)=480+0+2\*12\*4=576

(A3)(A4\*A5)=0+2880+20\*12\*60=17280 (A3\*A4)(A5)=960+0+20\*4\*60=5760

(A0)(A1\*A2\*A3)=0+600+10\*5\*12=1200 (A0\*A1)(A2\*A3)=100+480+10\*2\*12=820

(A0\*A1\*A2)(A3)=500+0+10\*20\*12=2900

(A1)(A2\*A3\*A4)=0+576+5\*2\*4=616 (A1\*A2)(A3\*A4)=200+960+5\*20\*4=1560

(A1\*A2\*A3)(A4)=600+0+5\*12\*4=840

(A2)(A3\*A4\*A5)=0+5760+2\*20\*60=8160 (A2\*A3)(A4\*A5)=480+0+2\*12\*60=1920

(A2\*A3\*A4)(A5)=576+0+2\*4\*60=1056

(A0)(A1\*A2\*A3\*A4)=0+616+10\*5\*4=816 (A0\*A1)(A2\*A3\*A4)=100+576+10\*2\*4=756

(A0\*A1\*A2)(A3\*A4)=500+960+10\*20\*4=2260 (A0\*A1\*A2\*A3)(A4)=820+0+10\*12\*4=1300

(A1)(A2\*A3\*A4\*A5)=0+1056+5\*2\*60=1656 (A1\*A2)(A3\*A4\*A5)=200+5760+5\*20\*60=11960

(A1\*A2\*A3)(A4\*A5)=600+2880+5\*12\*60=7080 (A1\*A2\*A3\*A4)(A5)=616+0+5\*4\*60=1816

(A0)(A1\*A2\*A3\*A4\*A5)=0+1656+10\*5\*60=4656

(A0\*A1)(A2\*A3\*A4\*A5)=100+1056+10\*2\*60=2356

(A0\*A1\*A2)(A3\*A4\*A5)=500+5760+10\*20\*60=18260

(A0\*A1\*A2\*A3)(A4\*A5)=820+2880+10\*12\*60=10900

(A0\*A1\*A2\*A3\*A4)(A5)=756+0+10\*4\*60=3156

optimal order: [(A0\*A1)]\*[(A2\*A3\*A4)\*(A5)]